

Study Guide

5th Grade AL Standards

01/23/2019

Multiply Whole No: 3-Digits by 2-Digits

Multiplying multiple-digit numbers often requires regrouping, also called carrying, trading, or renaming. Regrouping occurs when the product of a column is equal to or greater than ten.

The following is an example of a multiple-digit multiplication problem.

Example: Multiply.
$$\begin{array}{r} 237 \\ \times 56 \\ \hline \end{array}$$

(1)	(2)	(3)	(4)	(5)
$\begin{array}{r} 237 \\ \times 56 \\ \hline 1422 \\ \hline \end{array}$	$\begin{array}{r} 237 \\ \times 56 \\ \hline 1422 \\ \hline \end{array}$	$\begin{array}{r} 237 \\ \times 56 \\ \hline 1422 \\ \hline \end{array}$	$\begin{array}{r} 237 \\ \times 56 \\ \hline 1422 \\ \hline \end{array}$	$\begin{array}{r} 237 \\ \times 56 \\ \hline 1422 \\ \hline \end{array}$

Step 1: Rewrite the problem.

Step 2: Multiply 237 by 6.

Instructions: Multiply 6 times 7 which equals 42. Write the 2 below the line in the ones column and carry over 4 to the top of the tens column. Multiply 6 times 3 and add the 4 you carried over earlier: (6

$\times 3) + 4 = 22$. Write the 2 below the line in the tens column and carry over the 2 to the top of the hundreds column. Multiply 6 times two and add the 2 you carried over earlier: (6 $\times 2) + 2 = 14$. Write 14 to the left of the numbers you have already written below the line. The product is 1,422.

Step 3: Write 0 in the ones column as a place holder below the product of Step 2.

Step 4: Multiply 237 by 5.

Instructions: Multiply 5 times 7 which equals 35. Write the 5 in the tens column below the product from Step 2, and carry over the 3 to the top of the tens column (cross out the 4 that is there from Step 2).

Multiply 5 times 3 and add the 3 you carried over earlier: (5 $\times 3) + 3 = 18$. Write the 8 in the hundreds column and carry over 1 to the top of the hundreds column (cross out the 2 that is there from Step 2).

Multiply 5 times 2 and add the 1 you carried over earlier: (5 $\times 2) + 1 = 11$. Write 11 to the left of the 8. The product is 11,850.

Step 5: Add the two products (1,422 + 11,850) to determine the answer. Insert a comma to the left of the hundreds place.

Answer: 13,272

To help reinforce the concept of multiplying three-digit numbers by two-digit numbers, create two sets of index cards. On the first set of index cards, write three-digit numbers (one number per card). On the second set of index cards, write two-digit numbers (one number per card). Leave the cards in two separate piles. Have the student draw one card from each pile and multiply the numbers.

Add Decimals: No Regrouping

Adding two decimal numbers with more than one digit is very similar to adding whole numbers. At this level,

problems are presented in vertical format with the decimal points lined up correctly. There is no regrouping, carrying, borrowing, or trading involved in this skill.

A decimal number is a number that uses place value and a decimal point to show tenths, hundredths, thousandths, etc. For example, the number 3.57 has a 5 in the tenths place and a 7 in the hundredths place.

The difference between adding decimals and adding whole numbers is the fact that the decimal points must be lined up before addition can occur. The following is a step-by-step example of adding decimals.

Example: Solve.

$$\begin{array}{r} 2.3 \\ + 1.4 \\ \hline \end{array}$$

(1)	(2)	(3)
2.3	2.3	2.3
+1.4	+1.4	+1.4
7	.7	3.7

Step 1: Add the tenths column ($3 + 4 = 7$). Write 7 in the tenths column (below the line).

Step 2: Bring down the decimal point.

Step 3: Add the ones column ($2 + 1 = 3$). Write 3 to the left of the decimal point to finish the problem.

The correct answer is $2.3 + 1.4 = 3.7$.

Add Decimals: Regrouping

Adding two decimal numbers with regrouping is very similar to adding whole numbers with regrouping. These problems are presented in a vertical format with the decimal points lined up correctly. These problems require regrouping, carrying, or trading.

A decimal number is a number that uses place value and a decimal point to show tenths, hundredths, thousandths, etc. For example, the number 3.57 has a 5 in the tenths place and a 7 in the hundredths place.

The difference between adding decimals with regrouping and adding whole numbers with regrouping is the fact that the decimal points must be lined up before addition can occur. The following is a step-by-step example of adding decimals with regrouping:

Example 1: Solve. $2.4 + 1.68 = ?$

(1)	(2)	(3)	(4)	(5)	(6)
2.4	2.40	2.40	2.40	2.40	2.40
+1.68	+1.68	+1.68	+1.68	+1.68	+1.68
8	08	08	08	.08	4.08

Step 1: Write the problem vertically. Make sure the decimal points are lined up.

Step 2: Add one zero to 2.4 so that the two numbers have the same number of digits.

Step 3: Add the hundredths column ($0 + 8 = 8$). Write the 8 in the hundredths column (below the line).

Step 4: Add the tenths column ($4 + 6 = 10$). Write the 0 in the tenths column (below the line) and carry the 1 to the ones column (left of the decimal point).

Step 5: Bring the decimal point straight down.

Step 6: Add the ones column, including the 1 that was carried ($1 + 2 + 1 = 4$). Place the 4 to the left of the decimal point to finish the problem.

The correct answer is $2.4 + 1.68 = 4.08$.

Example 2: Solve. $3.2 + 2.8 = ?$

(1)	(2)	(3)	(4)
3.2	3.2	3.2	3.2
+ 2.8	+ 2.8	+ 2.8	+ 2.8
6.0	6.0	6.0	6.0

Step 1: Write the problem vertically. Make sure the decimal points are lined up.

Step 2: Add the tenths column ($2 + 8 = 10$). Write the 0 in the tenths column (below the line) and carry the 1 to the ones column (left of the decimal point).

Step 3: Bring the decimal point straight down.

Step 4: Add the ones column, including the 1 that was carried ($1 + 3 + 2 = 6$). Place the 6 to the left of the decimal point to finish the problem.

The correct answer is $3.2 + 2.8 = 6.0$.

Subtract Decimals: No Regrouping

Subtracting two decimal numbers with more than one digit is very similar to subtracting whole numbers. These problems are presented in vertical format with the decimal points lined up correctly. There is no regrouping, carrying, borrowing, or trading involved in this skill.

A decimal number is a number that uses place value and a decimal point to show tenths, hundredths, thousandths, etc. For example, the number 3.57 has a 5 in the tenths place and a 7 in the hundredths place.

The difference between subtracting decimals and subtracting whole numbers is the fact that the decimal points must be lined up before subtraction can occur. The following is a step-by-step example of subtracting decimals without regrouping.

Example 1: $2.56 - 1.12 = ?$

(1)	(2)	(3)	(4)	(5)
2.56	2.56	2.56	2.56	2.56
- 1.12	- 1.12	- 1.12	- 1.12	- 1.12
1.44	1.44	1.44	1.44	1.44

Step 1: Make sure the decimal points are lined up.

Step 2: Subtract the hundredths column ($6 - 2 = 4$). Write 4 in the hundredths column.

Step 3: Subtract the tenths column ($5 - 1 = 4$). Write 4 in the tenths column.

Step 4: Bring down the decimal point.

Step 5: Subtract the whole numbers ($2 - 1 = 1$). Write 1 to the left of the decimal point to finish the problem.

The correct answer is $2.56 - 1.12 = 1.44$.

Subtract Decimals: Regrouping

Subtracting two decimal numbers with more than one digit is very similar to subtracting whole numbers. These problems are presented in a vertical format with the decimal points lined up correctly. These problems require regrouping, carrying, or trading.

A decimal number is a number that uses place value and a decimal point to show tenths, hundredths, thousandths, etc. For example, the number 3.57 has a 5 in the tenths place and a 7 in the hundredths place. A whole number's decimal point comes after the number. For example, $64 = 64.0$.

The difference between subtracting decimals with regrouping and subtracting whole numbers with regrouping is the fact that the decimal points must be lined up before subtraction can occur. The following is a step-by-step example of subtracting decimals with regrouping.

Example 1: Solve $3.40 - 0.85 = ?$

(1)	(2)	(3)	(4)	(5)
3.40	3. ¹ 4 ⁰	² 3. ¹ 4 ⁰	² 3. ¹ 4 ⁰	² 3. ¹ 4 ⁰
-0.85	-0.85	-0.85	-0.85	-0.85
	5	55	.55	2.55

Step 1: Make sure the decimal points are lined up.

Step 2: Begin with the hundredths column (two places to the right of the decimal point). Regrouping must occur because you cannot subtract 5 from 0. Borrow 1 whole from the next column, changing the 4 to a 3. Give the 1 to the hundredths column, creating 10. Subtract the hundredths column ($10 - 5 = 5$). Put the 5 in the hundredths column below the line.

Step 3: Subtract the tenths column. Regrouping must occur because you cannot subtract 8 from 3. Borrow 1 whole from the ones column (one place to the left of the decimal point), changing the 3 to a 2. Give the 1 to the tenths column. Subtract the tenths column ($13 - 8 = 5$). Put the 5 in the tenths column below the line.

Step 4: Bring down the decimal point.

Step 5: Subtract the ones column ($2 - 0 = 2$). Put the 2 to the left of the decimal point (below the line) to finish the problem.

The correct answer is $3.4 - 0.85 = 2.55$.

Add Decimals: Story Problems - A

Story problems, also called word problems, relate decimal numbers to actual situations. For example, if Sara weighs 32.5 pounds and Kimberly weighs 31.2 pounds, how much do Sara and Kimberly weigh together? The student must determine that addition is required to solve this problem. (Answer: 63.7 pounds)

Story problems are often very difficult for children to master. It may be beneficial for you to verify that the student is comfortable with addition and subtraction skills, as well as with reading skills. Relate word problems with everyday events.

Example 1: A recipe calls for 2.55 cups of flour and another recipe calls for 1.25 cups of flour. How many cups of flour do you need to make both recipes?

(1)	(2)	(3)	(4)	(5)
$\begin{array}{r} 2.55 \\ +1.25 \\ \hline \end{array}$	$\begin{array}{r} 2.55 \\ +1.25 \\ \hline 0 \end{array}$	$\begin{array}{r} 2.55 \\ +1.25 \\ \hline 80 \end{array}$	$\begin{array}{r} 2.55 \\ +1.25 \\ \hline .80 \end{array}$	$\begin{array}{r} 2.55 \\ +1.25 \\ \hline 3.80 \end{array}$

Step 1: Rewrite the problem vertically. Always line up the decimal points.

Step 2: Add the numbers in the hundredths column ($5 + 5 = 10$). Write the 0 in the hundredths position. Carry the 1 to the tenths column.

Step 3: Add the numbers in the tenths column, including the number carried over from the previous column ($1 + 5 + 2 = 8$). Write the 8 in the tenths position.

Step 4: Bring the decimal point down.

Step 5: Add the numbers in the ones position ($2 + 1 = 3$). Write the 3 to the left of the decimal point.

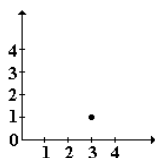
Answer: You need 3.80 cups of flour for the two recipes.

Coordinate Geometry - A

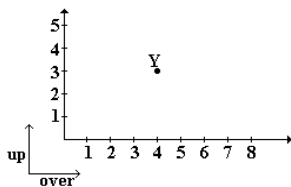
A coordinate graph is used to name the positions of objects placed on the graph.

It may be helpful to use graph paper to develop a coordinate graph. Help the student plot points on the graph and determine the coordinate pair.

Example 1: Plot a point that is 3 units over and 1 unit up from the zero. The coordinate points would be (3, 1). Remember that the horizontal position (or "over") is listed first in the coordinate pair, while the vertical position (or "up") is listed second.



Example 2: Where is the point Y?



Answer: Over 4, up 3 (always go over first, and then up)

Order of Operations with Decimals - A

Performing operations with decimals is similar to performing operations with whole numbers.

Operations inside parentheses are performed first.

Example 1: $(7.2 + 3.4) + (2.31 + 5.352) = ?$

(1) $7.2 + 3.4 = 10.6$ and $2.31 + 5.352 = 7.662$

(2) $10.6 + 7.662 = ?$

(3) $10.6 + 7.662 = 18.262$

Step 1: Perform all operations within parentheses. Add $7.2 + 3.4 = 10.6$. And add $2.31 + 5.352 = 7.662$.

Step 2: Rewrite the equation with the new numbers in place of the parentheses.

Step 3: Add 10.6 and 7.662.

The answer is 18.262

Example 2: $(13.295 - 1.62) - (3.5625 + 5.92) = ?$

(1) $13.295 - 1.62 = 11.675$ and $3.5625 + 5.92 = 9.4825$

(2) $11.675 - 9.4825 = ?$

(3) $11.675 - 9.4825 = 2.1925$

Step 1: Perform operations within parentheses. Subtract $13.295 - 1.62 = 11.675$. And add $3.5625 + 5.92 = 9.4825$.

Step 2: Rewrite the equation with the new numbers in place of the parentheses.

Step 3: Subtract 9.4825 from 11.675.

The answer is 2.1925.

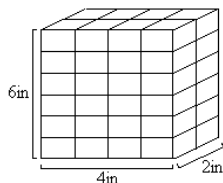
Volume - A

Volume is the measurement of the interior space of a three-dimensional figure. Volume is measured in cubic units.

The formula for calculating the volume of a rectangular solid is length multiplied by width multiplied by height.

$$\text{Volume} = \text{length} \times \text{width} \times \text{height}$$

Example 1: A figure has a length = 2 inches, a width = 4 inches, and a height = 6 inches. What is the volume of the figure?



Solution:

The volume of the figure is the length multiplied by the width multiplied by the height.

$$V = 2 \times 4 \times 6 = 48$$

Answer: 48 cubic inches.

It may be helpful to develop a series of problems and help the student determine the correct volume measurement.

Units of Measurement - B

Students must convert various forms of measurement, specifically U.S. Standard (also called Customary Units) and the Metric system.

The following shows the relationship between the Customary Units of length:

12 inches = 1 foot
3 feet = 1 yard
36 inches = 1 yard
5,280 feet = 1 mile
1,760 yards = 1 mile

The following shows the relationship between the Metric Units of length:

10 millimeters = 1 centimeter
100 millimeters = 1 decimeter
1,000 millimeters = 1 meter
10 centimeters = 1 decimeter
100 centimeters = 1 meter
1,000 meters = 1 kilometer
100 meters = 1 hectometer
10 meters = 1 dekameter

Use the above relationships to convert measurement units in real life situations. For example, a table is 36 inches tall. There are 12 inches in 1 foot. Divide 36 by 12, the result is 3. The table is 3 feet tall. Continue with other objects and also the metric system.

Example: Convert 189 inches into feet.

- (1) $189 \div 12 = 15.75$
- (2) $12 \times 0.75 = 9$
- (3) 15 feet 9 inches

Step 1: Divide 189 inches by 12 because there are 12 inches in one foot.

Step 2: Multiply 12 inches by 0.75 because we need to determine how many inches are in 0.75 (or 3/4) feet.

Step 3: Put the number of feet together with the number of inches that are left over.

Answer: 15 feet 9 inches

Add Decimals: Hundredths

Adding two decimal numbers with more than one digit (columns of numbers) is very similar to adding whole

numbers. Like whole numbers, addition of decimals often requires regrouping (carrying, trading, renaming). Regrouping occurs when the total of the numbers in a column (i.e., ones position) is equal to or greater than ten. Problems are presented in both vertical and horizontal formats.

The following is a step-by-step example of a problem that requires regrouping:

Example 1: Solve. $8.97 + 5.36 = ?$

(1)	(2)	(3)	(4)
8.97	8.97	8.97	8.97
$+ 5.36$	$+ 5.36$	$+ 5.36$	$+ 5.36$
-----	-----	-----	-----
	3	33	14.33

Step 1: Rewrite the problem vertically. Always line up the decimal points.

Step 2: Add the numbers in the hundredths position ($7 + 6 = 13$). Write the 3 in the hundredths position. Carry the 1 to the next column (tenths).

Step 3: Add the numbers in the tenths column, including the number carried over from the previous column ($1 + 9 + 3 = 13$). Write the 3 in the tenths position. Carry the 1 to the next column (ones). Bring the decimal point down.

Step 4: Add the numbers in the ones position, including the number carried over from the previous column ($1 + 8 + 5 = 14$). Write the 14 to the left of the decimal point.

Answer: $8.97 + 5.36 = 14.33$

Example 2: Solve. $\$23.91 + \$32.64 = ?$

(1)	(2)	(3)	(4)	(5)	(6)
\$23.91	\$23.91	\$23.91	\$23.91	\$23.91	\$23.91
$+ \$32.64$	$+ \$32.64$	$+ \$32.64$	$+ \$32.64$	$+ \$32.64$	$+ \$32.64$
-----	-----	-----	-----	-----	-----
	5	55	.55	6.55	\$56.55

Step 1: Rewrite the problem vertically. Always line up the decimal points.

Step 2: Add the numbers in the hundredths position ($1 + 4 = 5$). Write the 5 in the hundredths position.

Step 3: Add the numbers in the tenths column ($9 + 6 = 15$). Write the 5 in the tenths position. Carry the 1 to the next column (ones).

Step 4: Bring the decimal point down.

Step 5: Add the numbers in the ones position, including the number carried over from the previous column ($1 + 3 + 2 = 6$). Write the 6 in the ones position.

Step 6: Add the numbers in the tens position ($2 + 3 = 5$). Write the 5 in the tens position. Bring down the dollar sign.

Answer: $\$23.91 + \$32.64 = \$56.55$

Subtract Decimals: Hundredths

Subtracting decimal numbers requires a strong understanding of the subtraction process, specifically regrouping (trading or borrowing).

The following is a step-by-step example of subtracting two decimal numbers.

Example 1: Solve: $6.92 - 2.47 = ?$

(1)	(2)	(3)	(4)	(5)
6.92	6. ⁸¹ 2	6. ⁸¹ 2	6. ⁸¹ 2	6. ⁸¹ 2
-2.47	-2.47	-2.47	-2.47	-2.47
	5	45	.45	4.45

Step 1: Rewrite the problem vertically. Always line up the decimal points.

Step 2: Subtract the hundredths column. Regrouping must be used because 7 can not be subtracted from 2. Borrow 1 from the next column (the tenths position), changing the 9 to an 8. Give the one to the hundredths column, creating a 12. Subtract 7 from 12 ($12 - 7 = 5$). Write the 5 in the hundredths column.

Step 3: Subtract the tenths column ($8 - 4 = 4$). Write the 4 in the tenths column.

Step 4: Bring the decimal point straight down.

Step 5: Complete the subtraction of the ones ($6 - 2 = 4$). Write the 4 in the ones position.

Answer: $6.92 - 2.47 = 4.45$.

Subtract Decimals: Story Problems - A

Story problems, also called word problems, relate subtraction of decimal numbers to actual situations.

Operational symbols, such as the subtraction (-) symbol, are replaced with text. Problems dealing with money are also included in this skill.

Story problems are often very difficult for students to master. It may be beneficial for you to confirm that the student is comfortable with subtraction skills.

Create equations that relate to his or her daily activities, such as sports or music lessons. Help the student determine the correct formulas.

Example 1: If Tania swam 7.9 miles and Pete swam 3.5 miles, how many more miles did Tania swim than Pete? (The student must determine that subtraction is required to perform this problem.)

(1)	(2)	(3)	(4)
7.9	7.9	7.9	7.9
-3.5	-3.5	-3.5	-3.5
	4	.4	4.4

Step 1: Write the problem vertically.

Step 2: Subtract the numbers in the tenths column ($9 - 5 = 4$). Write the 4 in the tenths column.

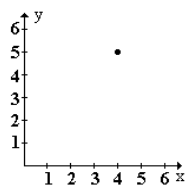
Step 3: Bring the decimal point down.

Step 4: Subtract the numbers in the ones column ($7 - 3 = 4$). Write the 4 to the left of the decimal point.

Answer: Tania swam 4.4 more miles than Pete.

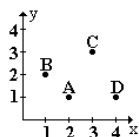
Coordinate Geometry - B

A coordinate graph is used to name the position of points. Horizontal positions are listed first in coordinate pairs and vertical positions are listed second. For example, the coordinate pair (4, 5) is at the horizontal position 4 and the vertical position 5.



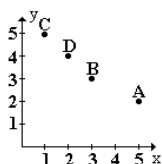
It may be helpful to use graph paper to develop a coordinate graph. Help the student plot points on the graph and determine the coordinate pair for each.

Example 1: What point has the number pair (2, 1)?



Answer: Point A because it is 2 units over and 1 unit up.

Example 2: What are the coordinates of Point C?



Answer: (1, 5) because Point C is 1 unit over and 5 units up.

Divide Whole No: 3+ Digits by 2-Digits

Dividing a three-digit whole number by a two-digit whole number usually requires using long division. A remainder is a number left over in a division problem. Remainders must be less than the divisor.

It may be helpful to verify that the student understands the format of long division. The following is a step-by-step example of a long division problem that has a remainder.

Solve: 2937 divided by 14.

(1)	(2)	(3)	(4)
$14 \overline{)2937}$	$14 \overline{)2937}$	$14 \overline{)2937}$	$14 \overline{)2937}$
	$\underline{28}$	$\underline{28}$	$\underline{28}$
	13	13	13
		$\underline{0}$	$\underline{0}$
		137	137
			$\underline{126}$
			11

Step 1: Write the problem in long division format.

Step 2: Find the quotient of 29 divided by 14 (2). Multiply 14 by 2 and write the product (28) below 29. Subtract 28 from 29 (1). Bring the 3 down next to the 1.

Step 3: Find the quotient of 13 divided by 14 (0). Multiply 14 by 0 and write the product (0) below the 13. Subtract 0 from 13 (13). Bring the 7 down next to the 13.

Step 4: Find the quotient of 137 divided by 14 (9). Multiply 14 by 9 and write the product (126) below the 137. Subtract 126 from 137 (11). Write the remainder (R11) next to the ones position.

The correct answer is 209 with a remainder of 11.

Units of Measurement - A

Students must use the relationships between measurements to determine equivalent units of measurement. They must also solve problems involving units of measurement.

The following shows the relationship between the Customary Units of length:

12 inches = 1 foot
3 feet = 1 yard
36 inches = 1 yard
5,280 feet = 1 mile
1,760 yards = 1 mile

The following shows the relationship between the Metric Units of length:

10 millimeters = 1 centimeter
100 millimeters = 1 decimeter
1,000 millimeters = 1 meter
10 centimeters = 1 decimeter
100 centimeters = 1 meter
1,000 meters = 1 kilometer
100 meters = 1 hectometer
10 meters = 1 dekameter

Use the above relationships to convert measurement units in real life situations. For example, a table is 36 inches tall. There are 12 inches in 1 foot. Divide 36 by 12, the result is 3. The table is 3 feet tall. Continue with other objects and also the metric system.

The student should also practice adding and subtracting different units of measurement. For example, a fence is 4 meters long. Jim put a gate on the end of the fence that was 20 decimeters long. How long is the fence and the gate combined?

- (1) 20 decimeters = 2 meters
(2) 4 meters + 2 meters = ?

Step 1: Convert units to like units.

Step 2: Add units

Answer: 6 meters

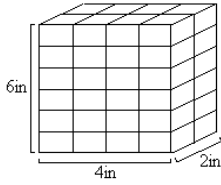
Volume - B

Volume is the measurement of a three-dimensional figure's interior space. Volume is measured in cubic units.

The formula for calculating the volume of a rectangular solid is length multiplied by width multiplied by height.

$$\text{Volume} = \text{length} \times \text{width} \times \text{height}$$

Example 1: A figure has a length = 2 inches, a width = 4 inches, and a height = 6 inches. What is the volume of the figure?



Solution:

The volume of the figure is the length multiplied by the width multiplied by the height.

$$V = 2 \times 4 \times 6 = 48$$

Answer: 48 cubic inches.

It may be helpful to develop a series of problems and help the student determine the correct volume measurement.

Mass/Capacity - A

Mass is the total amount of matter that a figure contains. Capacity is the liquid volume of a figure. Weight is the quantity which an object weighs.

A creative method for improving the student's understanding of measurement is to utilize actual objects. For example, help the student determine the approximate weight of objects in the home. For capacity, use actual figures, such as glasses and other containers. Help the student determine the capacity of liquid in each container.

g = grams

1 milligram (mg) = 0.001 g

1 centigram (cg) = 0.01 g

1 decigram (dg) = 0.1 g

1 dekagram (dag) = 10 g

1 hectogram (hg) = 100 g

1 kilogram (kg) = 1,000 g

1 metric ton (t) = 1,000 kg

The relationships in this chart are the same for liters, except for the metric ton relationship.

U.S. Customary System of Measurement:

16 ounces (oz) = 1 pound (lb)

16 cups (C) = 1 gallon (gal)

8 pints (pt) = 1 gallon (gal)

4 quarts (qt) = 1 gallon (gal)

Add Decimals: Story Problems - B

Story problems, also called word problems, relate addition of decimal numbers to actual situations. Operational symbols, such as the addition (+) symbol, are replaced with text. Word problems in this skill also deal with money (\$3.74).

Story problems are often very difficult for students to master. It may be beneficial for you to create humorous problems and help the student determine the correct formulas.

For example, Fred ran 8.971 miles on Saturday and 5.363 miles on Sunday. How many miles did Fred run in all?

(1)	(2)	(3)	(4)	(5)
8.971	8.971	$\overset{1}{8.971}$	$\overset{11}{8.971}$	$\overset{11}{8.971}$
+ 5.363	+ 5.363	+ 5.363	+ 5.363	+ 5.363
	4	34	.334	14.334

Step 1: Rewrite the problem vertically. Always line up the decimal points.

Step 2: Add the numbers in the thousandths position ($1 + 3 = 4$). Write the 4 in the thousandths position.

Step 3: Add the numbers in the hundredths position ($7 + 6 = 13$). Write the 3 in the hundredths position. Carry the 1 to the next column (tenths).

Step 4: Add the numbers in the tenths column, including the number carried over from the previous column ($1 + 9 + 3 = 13$). Write the 3 in the tenths position. Carry the 1 to the next column (ones). Bring the decimal point down.

Step 5: Add the numbers in the ones position, including the number carried over from the previous column ($1 + 8 + 5 = 14$). Write the 14 to the left of the decimal point.

Answer: Fred ran 14.334 miles.

Subtract Decimals: Story Problems - B

Story problems, also called word problems, relate subtraction of decimal numbers to actual situations. Operational symbols, such as the subtraction (-) symbol, are replaced with text. Problems dealing with money are also included in this skill.

Story problems are often very difficult for students to master. It may be beneficial for you to confirm that the student is comfortable with subtraction skills.

Create equations that relate to his or her daily activities, such as sports or music lessons. Help the student determine the correct formulas.

For example, if Ezekiel swam 7.92 miles and Parker swam 3.54 miles, how many more miles did Ezekiel swim than Parker? The student must determine that subtraction is required to perform this problem

(1)	(2)	(3)	(4)	(5)
7.92	$7.\overset{8}{9}2$	$7.\overset{8}{9}2$	$7.\overset{8}{9}2$	$7.\overset{8}{9}2$
- 3.54	- 3.54	- 3.54	- 3.54	- 3.54
	8	38	.38	4.38

Step 1: Write the problem vertically. Make sure the decimal points are lined up.

Step 2: Begin with the hundredths column (two places to the right of the decimal point). Regrouping

must occur because you cannot subtract 4 from 2. Borrow 1 whole from the tenths column (one place to the right of the decimal point), changing the 9 to an 8. Give the 1 to the hundredths column, creating 12. Subtract the hundredths column ($12 - 4 = 8$). Put the 8 in the hundredths column.

Step 3: Subtract the tenths column. ($8 - 5 = 3$) Put the 3 in the tenths column.

Step 4: Bring down the decimal point.

Step 5: Subtract the ones column ($7 - 3 = 4$). Put the 3 to the left of the decimal point to finish the problem.

Answer: Ezekiel swam 4.38 more miles than Parker.

Multiply Decimals: Hundredths

Multiplying a decimal number by another decimal number (5.39×4.3) requires a strong understanding of multiplication skills, specifically with multiple digit numbers.

The following is a step-by-step example of a decimal number multiplied with another decimal number.

Solve: $5.39 \times 4.3 = ?$

(1)	(2)	(3)	(4)
5.39	5.39	5.39	5.39
$\times 4.3$	$\times 4.3$	$\times 4.3$	$\times 4.3$
-----	-----	-----	-----
1617	1617	1617	1617
	$+21560$	$+21560$	$+21560$
	-----	-----	-----
	23177	23177	23177

Step 1: Rewrite horizontal problems vertically.

Step 2: Multiply 539 by 3. Write the product (1617) below the line.

Step 3: Place a 0 below the product of Step 2 in the ones position. Multiply 539 by 4. Write the product (2156) to the left of the 0. Add the two products ($1617 + 21560$) and write the sum (23177) below the line.

Step 4: Place the decimal point. Each place to the right of the decimal point is a decimal place. Count the number of decimal places in the factors (3). Place the decimal point in that position in the product. The number of decimal places in the product equals the sum of the decimal places in the factors.

The correct answer is $5.39 \times 4.3 = 23.177$

Add Fractions: Different Denominator

A fraction is comprised of two parts: a numerator (the top number) and a denominator (the bottom number). For example, in the fraction $2/3$, the "2" is the numerator and the "3" is the denominator. In order to add fractions, you must have a common denominator. A common denominator is a whole number that is a common multiple of the denominators of two or more fractions. For example, 12 and 24 are both common denominators for $3/4$ and $5/6$, because 4 and 6 will both divide into 12 and into 24.

The following is a step-by-step example of adding two fractions with different denominators.

Example 1: Reduce all fractions to lowest terms.

$$\begin{array}{r} \frac{1}{5} + \frac{4}{9} = ? \\ \text{(1)} \qquad \text{(2)} \qquad \text{(3)} \qquad \text{(4)} \\ \frac{1}{5} \qquad \frac{1 \times 9}{5 \times 9} \qquad \frac{9}{45} \qquad \frac{9}{45} \\ + \frac{4}{9} \qquad + \frac{4 \times 5}{9 \times 5} \qquad + \frac{20}{45} \qquad + \frac{20}{45} \\ \hline \qquad \qquad \qquad \frac{29}{45} \end{array}$$

Step 1: Rewrite horizontal problems vertically. (This step is not necessary, but many students find it easier to add fractions when the problems are written vertically.)

Step 2: Find a common denominator (a common multiple of the denominators of two or more fractions). For this problem, the common denominator is 45, because 9 and 5 will both divide into 45.

Step 3: Multiply $1/5$ by $9/9$. Rewrite the first fraction as $9/45$. Multiply $4/9$ by $5/5$. Rewrite the second fraction as $20/45$.

Step 4: Add the numerators together ($9 + 20 = 29$). The denominator (45) remains the same.

Answer: $\frac{1}{5} + \frac{4}{9} = \frac{29}{45}$.

It may be necessary to reduce a fraction that is part of an answer. A fraction is in lowest terms when the numerator and denominator do not have a common factor greater than one. To reduce a fraction, determine the largest number that the numerator and the denominator can both be divided by and divide them by that number.

Example 2: Reduce all fractions to lowest terms.

$$\frac{32}{40} \\ \frac{32 \div 8}{40 \div 8} = \frac{4}{5}$$

Solution: The largest number 32 and 40 can both be divided by is 8. Divide 32 by 8 and divide 40 by 8.

The correct answer is: $\frac{32}{40}$ can be reduced to $\frac{4}{5}$.

An improper fraction is a fraction in which the numerator is greater than or equal to the denominator. All improper fractions can be rewritten as mixed numbers or as whole numbers. The following is an example of how to write an improper fraction as a mixed number.

Example 3: Reduce all fractions to lowest terms.

$$\frac{55}{18} \\ \frac{55}{18} = 3 \frac{1}{18}$$

Solution: 18 will divide into 55 three times ($18 \times 3 = 54$) with one left over. Write the 3 as a whole number and then make the fraction $1/18$.

The correct answer is: $\frac{55}{18}$ can be written as $3 \frac{1}{18}$.

Add Fractions: Story Problems - A

Story problems, also called word problems, relate addition of fractions to actual situations. Operational symbols, such as the addition (+) symbol, are replaced with text.

Example 1: If Ellen ate $\frac{2}{3}$ of a pie and Jim ate $\frac{1}{8}$ of a pie, how much pie did Ellen and Jim eat all together?

If Ellen ate $\frac{2}{3}$ of a pie and Jim ate $\frac{1}{8}$ of a pie, how much pie did Ellen and

(1)	(2)	(3)
$\frac{2}{3}$	$\frac{2}{3} \times \frac{8}{8}$	$\frac{16}{24}$
$+\frac{1}{8}$	$+\frac{1}{8} \times \frac{3}{3}$	$+\frac{3}{24}$
		19
		24

Step 3: Rewrite the problem with the fractions having common denominators. Add the numerators of the fractions ($16 + 3 = 19$). The denominator (24) remains the same.

Answer: Jim and Ellen ate $\frac{19}{24}$ of a pie.

Colin ran $\frac{2}{3}$ of a mile on Monday. On Tuesday, he ran another $\frac{2}{3}$ of a

Example 2: mile. How many miles did Colin run in all?

(1)	(2)	(3)
$\frac{2}{3}$	$\frac{2}{3}$	$\frac{4}{3} = 1\frac{1}{3}$
$+\frac{2}{3}$	$+\frac{2}{3}$	
$\frac{4}{3}$	$\frac{4}{3}$	

Step 3: The improper fraction $\frac{4}{3}$ needs to be written as a mixed number. Three will divide into 4 one time with 1 left over. The whole number is 1 and the fraction is $\frac{1}{3}$.

Answer: Colin ran a total of $1\frac{1}{3}$ miles.

A fraction is comprised of two parts: a numerator (the top number) and a denominator (the bottom number). For example, in the fraction $\frac{2}{3}$ the "2" is the numerator and the "3" is the denominator. In order to subtract fractions, you must have a common denominator. A common denominator is a number that is a common multiple of the denominators of two or more fractions. For example, 12 and 24 are both common denominators for $\frac{3}{4}$ and $\frac{5}{6}$, because 4 and 6 will both divide into 12 and 24.

The following is a step-by-step example of subtracting two fractions with different denominators:

Solve:

$$\frac{6}{7} - \frac{3}{4} = ?$$

(1)	(2)	(3)	(4)
$\frac{6}{7}$	$\frac{6 \times 4}{7 \times 4}$	$\frac{24}{28}$	$\frac{24}{28}$
$-\frac{3}{4}$	$-\frac{3 \times 7}{4 \times 7}$	$-\frac{21}{28}$	$-\frac{21}{28}$
			$\frac{3}{28}$

Step 1: Rewrite the problem vertically.

Step 2: Find a common denominator (a common multiple of the denominators). For this problem, the common denominator is 28. Multiply $\frac{6}{7}$ by $\frac{4}{4}$. Multiply $\frac{3}{4}$ by $\frac{7}{7}$.

Step 3: Rewrite $\frac{6}{7}$ as $\frac{24}{28}$. Rewrite $\frac{3}{4}$ as $\frac{21}{28}$.

Step 4: Subtract the numerators ($24 - 21 = 3$). The denominator (28) remains the same.

The correct answer is $\frac{6}{7} - \frac{3}{4} = \frac{3}{28}$.

Compare Decimals - C

Comparing decimal numbers involves determining which decimal in a set is either the least or the greatest. Real world problems, also called word or story problems, present decimal problems in text format. Students are required to read passages and determine the questions being asked. They should then identify the elements needed to solve the problem, decide on the correct method to solve each problem, and find a solution.

Story problems are often difficult for students to master. It may be beneficial to confirm that the student is comfortable with his or her ability to compare decimals outside of a story problem context. To review this concept with the student, use the example below. Remember, when comparing numbers, line the decimal points up and compare the corresponding columns beginning with the highest place value.

Comparing Decimals Without a Context:

Example 1:

Put the decimals below in order from least to greatest:

2.4091, 2.904, 2.09, 2.049

Solution:

Step 1: Write the decimals in vertical form, lining up the decimal points.

2.4091
2.904
2.09
2.049

Step 2: Look at the numbers from left to right, stopping when any of the numbers are different. The two is the same in all of the numbers. In the tenths place (to the right of the decimal point), there is 4, 9, 0, and 0.

Step 3: The greatest number is 9, so 2.904 is the greatest number in the set. The next highest is the 4, so 2.4091 is next.

Step 4: Look at the two numbers with a 0 in the tenths place. In the hundredths place, one number has a 9 and the other number has a 4. Therefore, 2.09 is the third greatest, and 2.049 is the least. The numbers in order from least to greatest are: 2.049, 2.09, 2.4091, and 2.904.

Answer: 2.049, 2.09, 2.4091, 2.904.

A common mistake that students make is to assume that the numbers after the decimal place follow the rules of counting numbers (1, 2, 3, etc.). Specifically, students assume that the more digits there are in the number, the greater the number is. For example, students know that 215 is greater than 23 because 215 has more digits than 23. However, with decimals, 0.215 is less than 0.23, even though 0.215 contains more digits.

Comparing Decimals in a Real World Context:

When the student appears to have mastered the skill of comparing decimals, increase the difficulty of the problem by putting the decimals in a real world context. It may be beneficial to create problems that relate to his or her daily activities, such as sports or measurements. Then, help the student determine the correct process to find the solution.

Example 2:

Kendall is trying to reduce his weight class for wrestling. He wants to determine what time of day he should be weighed so that he will weigh the least. He weighed himself throughout the day and his results are in the table below.

<u>Time:</u>	<u>Weight (in kg):</u>
8:00 A.M.	70.3089
11:30 A.M.	70.3068
2:30 P.M.	70.31
5:15 P.M.	70.307

At what time did Kendall weigh the least?

Solution:

Step 1: Line up all of the decimal points and move from left to right. The numbers are all identical until the hundredths place. That is, the 70.3 part of the number is common to all of the numbers.

Step 2: Look at the hundredths place. There are three numbers with a 0 and one number with a 1 in the hundredths place. Therefore, the 70.31 is the greatest number, and it is eliminated because the question is asking for the smallest weight.

Step 3: Look at the thousandths place since all of the other numbers have a 0 in the hundredths place. The thousandths place contains a 6, a 7, and an 8 for the remaining numbers. Regardless of the number that follows it, 6 is the smallest number of the three that are remaining, so 70.3068 kg is the least Kendall weighed during the day.

Answer: Kendall weighed the least at 11:30 A.M.

Example 3:

Naomi's baseball team has won more games than it has lost for four years in a row. Her team has had almost the same record every year as shown in the table below.

<u>Year:</u>	<u>Record (Wins / No. of Games):</u>
1999	0.633
2000	0.64
2001	0.6398
2002	0.624

In what year was the team's record the highest?

Solution:

Step 1: Line up all of the decimal points and move from left to right. The numbers are all identical until the hundredths place. That is, the 0.6 part of the number is common to all of the numbers.

Step 2: Look at the numbers in the hundredths place. There are two numbers with a 3, one with a 2, and one with a 4 in the hundredths place. Therefore, the highest number is 0.64 and the year it occurred was 2000.

Answer: The team's record was the highest in 2000.

NOTE: When working with real world problems that involve speed, the lowest decimal number is equal to the fastest speed.

An activity relating to this skill would be to choose a certain stock from the newspaper or on the Internet (try a search for "stock quotes" and look for one that carries the decimal places out to the hundredths place) and monitor the stock once a day for one week. Determine which day the stock was worth the most and which day it was worth the least. A similar activity would be to have the student track his or her favorite professional sport team's record throughout the season or from year to year.

Subtract Fractions: Story Problems - A

In this skill, the story problems, also called word problems, relate subtraction of fractions to actual situations. Operational symbols, such as the subtraction (-) symbol, are replaced with text. The problems in this skill set do not require the students to reduce.

Story problems are often difficult for students to master. It may be beneficial for you to confirm that the student is comfortable with subtraction involving fractions outside of a story context first. The example below shows how to subtract two fractions without reducing.

Example 1:

Subtract.

$$\frac{3}{4} - \frac{5}{8} =$$

$$(1) \quad \frac{3}{4} - \frac{5}{8} =$$

$$\left(\frac{3 \times 2}{4 \times 2} \right) - \frac{5}{8} =$$

$$\frac{6}{8} - \frac{5}{8} =$$

$$(2) \quad \frac{6}{8} - \frac{5}{8} = \frac{1}{8}$$

Step 1: Find a common denominator (the least common multiple of the denominators of two or more fractions) for the two fractions. For this problem, a common denominator is 8 because 8 and 4 are both factors of 8 (they will both divide into 8 without remainders). Multiply $\frac{3}{4}$ by $\frac{2}{2}$ to get $\frac{6}{8}$.

Step 2: Subtract the numerators of the fractions ($6 - 5 = 1$). The denominator, 8, remains the same.

Answer: $\frac{1}{8}$

Subtracting Fractions Within Word Problems:

Once the student is comfortable with the skills involved in subtracting fractions, he or she should be ready to subtract fractions within word problems.

Example 2:

Kevin drank $\frac{3}{4}$ of a gallon of punch. Leia drank $\frac{1}{5}$ of a gallon of punch.

How much more punch did Kevin drink?

$$(1) \quad \frac{3}{4} - \frac{1}{5} =$$

$$(2) \quad \left(\frac{3 \times 5}{4 \times 5} \right) - \left(\frac{1 \times 4}{5 \times 4} \right) =$$

$$\frac{15}{20} - \frac{4}{20} =$$

$$(3) \quad \frac{15}{20} - \frac{4}{20} = \frac{11}{20}$$

Step 1: Determine that subtraction is required to solve the problem and write the needed equation.

Since Kevin drank $\frac{3}{4}$ of a gallon and Leia drank $\frac{1}{5}$ of a gallon, the equation will be $\frac{3}{4} - \frac{1}{5}$.

Step 2: Find a common denominator for the two fractions. For this problem, a common denominator is 20 because 4 and 5 will both divide into 20. Multiply $\frac{3}{4}$ by $\frac{5}{5}$ to get $\frac{15}{20}$. Multiply $\frac{1}{5}$ by $\frac{4}{4}$ to get $\frac{4}{20}$.

Step 3: Subtract the numerators of the fractions ($15 - 4 = 11$). The denominator, 20, remains the same.

Answer: Kevin drank $\frac{11}{20}$ of a gallon more than Leia.

An activity to reinforce the concept is to create story problems that relate to the student's daily activities, such as sports or music lessons. Help the student determine the correct answers to the story problems.

Subtract Fractions: Mixed Numbers - B

This study guide will focus on subtracting mixed fractions that require regrouping (borrowing). At this level, students will not be required to reduce their answers.

To review, a mixed number is a whole number followed by a fraction that, together, represent one value. If the numerator (the top number) of a fraction is less than the denominator (the bottom number), the fraction is called a proper fraction. If the numerator is equal to or greater than the denominator of a fraction, the fraction is called an improper fraction. An improper fraction can be rewritten as a mixed number. For example, $\frac{5}{3}$ is an improper fraction. It can be rewritten as $1 \frac{2}{3}$, which is a mixed number. The following is a step-by-step example of subtracting two mixed numbers when the second fraction is larger than the first and regrouping is necessary.

Example 1: Subtract.

$$\begin{array}{r} 5 \frac{2}{3} \\ - 2 \frac{3}{4} \\ \hline \end{array}$$

(1)	(2)	(3)	(4)	(5)	(6)
$5 \frac{2}{3}$	$5 \frac{2}{3}$	$5 \frac{2 \times 4}{3 \times 4}$	$5 \frac{8}{12}$	$\overset{4}{\cancel{5}} \frac{8}{12} + \frac{12}{12}$	$4 \frac{20}{12}$
$- 2 \frac{3}{4}$	$- 2 \frac{3}{4}$	$- 2 \frac{3 \times 3}{4 \times 3}$	$- 2 \frac{9}{12}$	$- 2 \frac{9}{12}$	$- 2 \frac{9}{12}$
					$2 \frac{11}{12}$

Step 1: Write the problem vertically.

Step 2: Separate the problem into subtraction of whole numbers and subtraction of fractions.

Step 3: Find a common denominator for the fractions. For this problem, the lowest common denominator is 12. Multiply $\frac{2}{3}$ by $\frac{4}{4}$ to get $\frac{8}{12}$. Multiply $\frac{3}{4}$ by $\frac{3}{3}$ to get $\frac{9}{12}$.

Step 4: Rewrite the problem so that the fractions have common denominators.

Step 5: Since the top fraction ($\frac{8}{12}$) is smaller than the bottom fraction ($\frac{9}{12}$), borrow 1, in the form of $\frac{12}{12}$, from the 5. Next, add $\frac{12}{12}$ to the $\frac{8}{12}$ to get $\frac{20}{12}$.

Step 6: Subtract the whole numbers ($4 - 2 = 2$). Subtract the numerators of the fractions ($20 - 9 = 11$). The denominator (12) remains the same.

Answer: $5\frac{2}{3} - 2\frac{3}{4} = 2\frac{11}{12}$

One way to reinforce this skill is to write mixed numbers on two sets of index cards. For one set, use large whole numbers (10 - 20) and small fractions (such as $\frac{2}{5}$ or $\frac{1}{8}$). For the other set, use small whole numbers (1 - 9) and large fractions (such as $\frac{3}{4}$ or $\frac{4}{5}$). Have the student draw one card from each set of cards and subtract the mixed number with the smaller whole number from the mixed number with the larger whole number. This could easily be made into a game either by making a game board or by borrowing one from another game. When a player gets an answer correct, he or she gets to move the number of spaces that matches the whole number in his or her answer. If there is no whole number in the player's correct answer, the player moves 1 space. The first person to make it to the end of the game board wins.

Multiply Fractions: Story Problems - A

In this skill, story problems, also called word problems, relate multiplication of fractions to actual situations. Operational symbols, such as the multiplication symbol, $\times$, are replaced with text. The problems in this skill set do not require the students to reduce.

Many students find story problems challenging. It may be useful to first confirm that students are comfortable multiplying fractions. Then, they can move on to problems that are presented in a story context.

The following is an example of the multiplication of two fractions that does not require reducing.

Example 1:

Multiply.

$$\frac{7}{9} \times \frac{2}{5} =$$

Solution: Multiply the numerators (the numbers on the top of the fraction), $7 \times 2 = 14$, and denominators (the numbers on the bottom of the fraction), $9 \times 5 = 45$. The solution is $\frac{14}{45}$.

Answer: $\frac{7}{9} \times \frac{2}{5} = \frac{14}{45}$

Multiplication of fractions, especially in the context of a story problem, can be confusing for students. It is important for them to remember that when multiplying two fractions between 0 and 1, the product is always a smaller fraction. This result is the opposite of what they are used to seeing with whole number multiplication.

The following is an example of the multiplication of two fractions, with no reducing required, in the context of a story problem.

Example 2:



The top snowball of Hanh's snowman was $\frac{2}{5}$ the size of the middle snowball and the middle snowball was $\frac{1}{3}$ the size of the bottom snowball. What fraction of the size of the bottom snowball was the top snowball?

(1) $\frac{1}{3} \times \frac{2}{5} =$

(2) $\frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$

Step 1: Determine that multiplication is required to solve the problem. To relate the bottom snowball to the top snowball, it is necessary to relate the bottom snowball to the middle snowball ($\frac{1}{3}$) and then the middle snowball to the top snowball ($\frac{2}{5}$). Multiply these two fractions to find the solution.

Step 2: Multiply the numerators, $1 \times 2 = 2$, and the denominators, $3 \times 5 = 15$. The solution is $\frac{2}{15}$.

Answer: $\frac{2}{15}$ of the size

An activity to reinforce this skill is to look at the hours the student spends each day on different activities. Break the day into three parts: the time spent in school, the time spent sleeping, and the time spent awake, but outside of school. Have the student determine the fraction of the entire day that he or she spends at school. Then have him or her determine the fraction of the time spent in school in math class. From this, ask the student to determine the fraction of his or her entire day that is spent in math class. For example, if the school day is 8 hours, he or she spends $\frac{8}{24}$ of the day in school. If math class is 1 hour long, $\frac{1}{8}$ of the time that he or she is in school is spent in math class. Finally, the student spends $\frac{8}{24} \times \frac{1}{8} = \frac{8}{192}$ of the entire day in math class. Repeat this process and ask the student how much of the time he or she spends awake and out of school is spent watching television and what fraction of the entire day that is.

Multiply Fractions: Story Problems - B

In this skill, story problems, also called word problems, relate multiplication of fractions to actual situations. Operational symbols, such as the multiplication symbol, $\times$, are replaced with text. The problems in this skill set require the students to reduce.

Many students find story problems challenging. It may be useful to first confirm that students are comfortable with multiplying fractions. Then, they can move on to problems that are presented in story problem context.

The following is an example of the multiplication of two fractions with no reducing required.

Example 1:

Multiply.

$$\frac{7}{9} \times \frac{2}{5} =$$

Solution: Multiply the numerators (the numbers on the top of the fraction), $7 \times 2 = 14$, and the denominators (the numbers on the bottom of the fraction), $9 \times 5 = 45$. The solution is $\frac{14}{45}$.

Answer: $\frac{7}{9} \times \frac{2}{5} = \frac{14}{45}$

The following is an example of the multiplication of two fractions with reducing required.

Example 2:

Multiply.

$$\frac{3}{10} \times \frac{5}{9} =$$

$$(1) \frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{10}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{9}}} =$$

$$(2) \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

Step 1: Use cross cancellation to reduce the fractions before multiplying. Look for any numerator that has a common factor with any denominator.

- In this example, 3 and 9 share the factor 3, so it can be divided into both numbers to reduce them.

Perform the divisions. $3 \div 3 = 1$ and $9 \div 3 = 3$.

- Also, 5 and 10 share the factor 5, so it can be divided into both numbers to reduce them. Perform the

divisions. $5 \div 5 = 1$ and $10 \div 5 = 2$.

Step 2: Multiply the numerators, $1 \times 1 = 1$, and the denominators, $2 \times 3 = 6$. The solution is $1/6$.

Answer: $\frac{1}{6}$

An alternative method when multiplying fractions that can be reduced is to multiply through as usual and look for common factors in the resulting fraction.

$$\begin{aligned}\frac{3}{10} \times \frac{5}{9} &= \frac{15}{90} \\ \frac{15 \div 15}{90 \div 15} &= \frac{1}{6}\end{aligned}$$

Multiplication of fractions, especially in the context of a story problem, can be confusing for students. It is important for them to remember that when multiplying two fractions between 0 and 1, the product is always a smaller fraction. This result is the opposite of what students are used to with whole number multiplication.

The following is an example of the multiplication of two fractions in the context of a story problem with no reducing required.

Example 3:



The top snowball of Hanh's snowman was $\frac{2}{5}$ the size of the middle snowball and the middle snowball was $\frac{1}{3}$ the size of the bottom snowball. What fraction of the size of the bottom snowball was the top snowball?

(1) $\frac{1}{3} \times \frac{2}{5} =$

(2) $\frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$

Step 1: Determine that multiplication is required to solve the problem. To relate the bottom snowball to the top snowball, it is necessary to relate the bottom snowball to the middle snowball ($1/3$) and then the middle snowball to the top snowball ($2/5$). Multiply these two fractions to find the solution.

Step 2: Multiply the numerators, $1 \times 2 = 2$, and the denominators, $3 \times 5 = 15$. The solution is $2/15$.

Answer: $\frac{2}{15}$ of the size

The following is an example of multiplying two fractions in the context of a story problem that requires reducing.

Example 4:

Carrie spent $\frac{1}{4}$ of her day at school. She spent $\frac{2}{7}$ of her school day at lunch and in between classes. How much of her entire day was spent at lunch and in between her classes at school?

(1) $\frac{1}{4} \times \frac{2}{7} =$

(2) $\frac{1}{\cancel{4}^2} \times \frac{\cancel{2}^1}{7} =$

(3) $\frac{1}{2} \times \frac{1}{7} = \frac{1}{14}$

Step 1: Determine that multiplication is required to solve the problem. To relate Carrie's whole day to her time spent at lunch and in between classes, it is necessary to relate her whole day to the school day ($1/4$) and then the school day to the time spent at lunch and in between classes ($2/7$). The solution needed is $2/7$ of $1/4$ of her school day. Multiply $2/7$ and $1/4$ to find the solution.

Step 2: Simplify the fractions using cross cancellation. The numerator 2 and the denominator 4 both have a factor of 2 in common. Perform the divisions. $2 \div 2 = 1$ and $4 \div 2 = 2$.

Step 3: Multiply the numerators, $1 \times 1 = 1$, and the denominators, $2 \times 7 = 14$. The solution is

1/14.

Answer: $\frac{1}{14}$ of her entire day

An activity to help reinforce this skill is to create scenarios with the student. These can be scenarios such as determining the fraction of the front lawn that has flower beds. Then, determining the fraction of the flower beds with a specific type of flower (such as daffodils). Finally, have the student determine the fraction of the front yard that has the specific type of flower (daffodils). For example, if $\frac{1}{8}$ of the front yard has flower beds and $\frac{2}{3}$ of the flower beds are filled with daffodils, then $\frac{2}{24}$ or $\frac{1}{12}$ of the entire front yard is a flower bed with daffodils.

Subtract Fractions: Mixed Numbers - A

This study guide will focus on subtracting mixed numbers that do not require regrouping (borrowing) or reducing.

To review, a mixed number is a whole number followed by a fraction that, together, represent one value. If the numerator (the top number) of a fraction is less than the denominator (the bottom number), the fraction is called a proper fraction. If the numerator is equal to or greater than the denominator of a fraction, the fraction is called an improper fraction. The following is a step-by-step example of how to subtract two mixed numbers with no regrouping or reducing.

Example 1: Subtract.

$$\begin{array}{r} 5\frac{3}{5} \\ -2\frac{1}{3} \\ \hline \end{array} \quad \begin{array}{l} (1) \\ (2) \end{array}$$
$$\begin{array}{r} 5\frac{3 \times 3}{5 \times 3} = 5\frac{9}{15} \\ 2\frac{1 \times 5}{3 \times 5} = 2\frac{5}{15} \\ \hline 3\frac{4}{15} \end{array}$$

Step 1: Find the lowest common denominator for the fractions (15) and rewrite the fractions using the common denominator. Multiply the numerator and denominator of $\frac{3}{5}$ by 3 (because $5 \times 3 = 15$) and multiply the numerator and denominator of $\frac{1}{3}$ by 5 (because $3 \times 5 = 15$).

Step 2: Subtract the new fractions. First, subtract the numerators of the fractions ($9 - 5 = 4$) and place the 4 in the numerator of the answer. The denominator remains the same (15). Now, subtract the whole numbers ($5 - 2 = 3$).

Answer: $3\frac{4}{15}$

One way to reinforce this skill is to write mixed numbers on two sets of index cards. For one set, use large whole numbers (10 - 20) and large fractions (such as $\frac{3}{4}$ or $\frac{4}{5}$). For the other set, use small whole numbers (1 - 9) and small fractions (such as $\frac{2}{5}$ or $\frac{1}{8}$). Have the student draw one card from each set of cards and subtract the mixed number with the smaller whole number from the mixed number with the larger whole number.

Multiply Whole No: 3+ Digits by 2-Digit

Multiplying multiple-digit numbers often requires regrouping (carrying, trading, renaming). Regrouping occurs when the product is equal to or greater than ten in a column.

The following is a step-by-step example of a multiple-digit multiplication problem.

Solve: $237 \times 56 = ?$

(1)	(2)	(3)	(4)	(5)
237	237	237	237	237
$\times 56$	$\times 56$	$\times 56$	$\times 56$	$\times 56$
-----	1422	1422	1422	1422
		0	11850	+11850
			-----	13272

Step 1: Rewrite the problem vertically.

Step 2: Multiply 237 by 6. Write the product (1422).

Step 3: Place a 0, as a place holder, below the product of Step 2 in the ones position.

Step 4: Multiply 237 by 5. Write the product (1185) to the left of the 0.

Step 5: Add the two products ($1422 + 11850$) to determine the answer. Insert a comma after the thousands place.

The correct answer is $237 \times 56 = 13,272$.

Order of Operations

Multiple-step equation problems test a student's understanding of multiple-step equations. Multiple-step equations must be performed in a specific order to solve the problems.

The order of operations must be followed when working with grouping symbols and/or multiple step operations. The order of operations are as follows:

1. Perform operations within parentheses, braces, or brackets.
2. Perform operations with exponents.
3. Multiply and divide from left to right
4. Add and subtract from left to right

It may be helpful to verify that the student understands the rules of multiple-step equations. Then, develop a series of multiple-step equations and help the student solve them.

Example 1: $8 + (9 \times 2) = ?$

- (1) $9 \times 2 = 18$
- (2) $8 + 18 = ?$
- (3) $8 + 18 = 26$

Step 1: Perform operations within the parentheses.

Step 2: Replace the expression in the parentheses with the product from Step 1.

Step 3: Complete the equation.

Answer: $8 + (9 \times 2) = 26$

Example 2: $[(8 + 7) \times 2] + 3 = ?$

- (1) $8 + 7 = 15$
- (2) $[15 \times 2] + 3 = ?$
- (3) $15 \times 2 = 30$
- (4) $30 + 3 = 33$

Step 1: Perform operations within the brackets. Within the brackets, perform operations within parentheses. $8 + 7 = 15$

Step 2: Rewrite the problem with the new value in place of the parentheses.

Step 3: Multiply 15 by 2 because according to the order of operations, multiplication and division are completed from left to right (the multiplication comes first in the problem). Now all operations within the brackets have been completed.

Step 4: Rewrite the problem with the new value in place of the multiplication problem. Divide 30 by 3 to get 10.

Answer: 10

Subtract Decimals: Story Problems - C

Story problems, also called word problems, relate subtraction of decimal numbers to actual situations.

Operational symbols, such as the subtraction (-) symbol, are replaced with text. Problems dealing with money are also included in this skill.

Story problems are often very difficult for students to master. It may be beneficial for you to confirm that the student is comfortable with subtraction skills.

Create equations that relate to his or her daily activities, such as sports or music lessons. Help the student determine the correct formulas.

Example 1: Tania swam 7.923 miles and Pete swam 3.547 miles. How many more miles did Tania swim than Pete? (The student must determine that subtraction is required to perform this problem).

(1)	(2)	(3)	(4)	(5)	(6)
$\begin{array}{r} 7.923 \\ -3.547 \\ \hline \end{array}$	$\begin{array}{r} 7.923 \\ -3.547 \\ \hline 6 \end{array}$	$\begin{array}{r} 7.923 \\ -3.547 \\ \hline 76 \end{array}$	$\begin{array}{r} 7.923 \\ -3.547 \\ \hline 376 \end{array}$	$\begin{array}{r} 7.923 \\ -3.547 \\ \hline .376 \end{array}$	$\begin{array}{r} 7.923 \\ -3.547 \\ \hline 4.376 \end{array}$

Step 1: Write the problem vertically. Make sure the decimal points are lined up.

Step 2: Begin with the thousandths column (three places to the right of the decimal point). Regrouping must occur because you cannot subtract 7 from 3. Borrow 1 whole from the hundredths column (two places to the right of the decimal point), changing the 2 to a 1. Give the 1 to the thousandths column, creating 13. Subtract the thousandths column ($13 - 7 = 6$). Put the 6 in the thousandths column.

Step 3: Subtract the hundredths column (two places to the right of the decimal point). Regrouping must occur because you cannot subtract 4 from 1. Borrow 1 whole from the tenths column (one place to the right of the decimal point), changing the 9 to an 8. Give the 1 to the hundredths column, creating 11. Subtract the hundredths column ($11 - 4 = 7$). Put the 7 in the hundredths column.

Step 4: Subtract the tenths column ($8 - 5 = 3$). Put the 3 in the tenths column.

Step 5: Bring down the decimal point.

Step 6: Subtract the ones column ($7 - 3 = 4$). Put the 3 to the left of the decimal point to finish the problem.

Answer: Tania swam 4.376 more miles than Pete.

Divide Decimals

Dividing a decimal number by another decimal number requires repositioning the decimal point so that it does not appear in the divisor.

The following is a step-by-step example of a decimal number divided by a decimal number.

Solve: 17.28 divided by 3.2 = ?

(1)	(2)	(3)
$\begin{array}{r} 3.2 \overline{)17.28} \end{array}$	$\begin{array}{r} 5. \\ 32 \overline{)172.8} \\ \underline{160} \\ 128 \\ \underline{128} \\ 0 \end{array}$	$\begin{array}{r} 5.4 \\ 32 \overline{)172.8} \\ \underline{160} \\ 128 \\ \underline{128} \\ 0 \end{array}$

Step 1: Write the problem in long division format.

Step 2: There is a decimal point in the divisor. Multiply the dividend and the divisor by 10, thus moving the decimal point one place to the right. Then, place the decimal point directly above the decimal point in the dividend.

Step 3: Division follows the same format as with whole numbers. 32 goes into 172 five times because $32 \times 5 = 160$. Place 5 in the ones position. Subtract 160 from 172 resulting in 12. Bring down the 8.

Step 4: 32 goes into 128 four times because $32 \times 4 = 128$. Place 4 in the tenths position. Subtract 128 from 128 resulting in zero.

The correct answer for 17.28 divided by 3.2 is 5.4.

Divide Decimals: Story Problems

Story problems, also called word problems, relate division of decimal numbers to actual situations.

Operational symbols, such as the division symbol, are replaced with text. This skill also includes problems dealing with money.

Story problems are often very difficult for students to master. It may be useful for you to confirm that the student is comfortable with division skills. Then, create equations that relate to his or her daily activities, such as sports or music lessons. Help the student determine the correct formulas.

Example 1: If there are 18.9 pies and 9 children, how many pies does each child receive? (The student must determine that division is required to perform this problem.)

(1)	(2)	(3)	(4)
$\begin{array}{r} 9 \overline{)18.9} \end{array}$	$\begin{array}{r} 2 \\ 9 \overline{)18.9} \\ \underline{-18} \\ 09 \end{array}$	$\begin{array}{r} 2. \\ 9 \overline{)18.9} \\ \underline{-18} \\ 09 \end{array}$	$\begin{array}{r} 2.1 \\ 9 \overline{)18.9} \\ \underline{-18} \\ 09 \\ \underline{-9} \\ 0 \end{array}$

Step 1: Write the problem in long division format.

Step 2: Division follows the same format as with whole numbers. 9 goes into 18 two times because $9 \times 2 = 18$. Place 2 in the ones position. Subtract 18 from 18 resulting in 0. Bring down the 9.

Step 3: Place the decimal point.

Step 4: 9 goes into 9 one time because $9 \times 1 = 9$. Place 1 in the tenths position. Subtract 9 from 9 resulting in 0.

The answer is each child receives 2.1 pies.

Add Fractions: Mixed Numbers - A

Adding mixed fractions requires a solid understanding of adding fractions and the multiplication table. If the numerator of a fraction is less than the denominator, the fraction is called a proper fraction. If the numerator is equal to or greater than the denominator, the fraction is called an improper fraction. An improper fraction can be rewritten as a mixed fraction. For example, $5/3$ is an improper fraction. It can be rewritten as $1\frac{2}{3}$, which is a mixed fraction.

The following is a step-by-step example of adding two mixed fractions with different denominators.

Example 1: Reduce all fractions to lowest terms.

$$3\frac{1}{5} + 6\frac{4}{9} =$$

(1)	(2)	(3)	(4)	(5)
$3\frac{1}{5}$	$3\frac{1}{5}$	$\frac{1}{5} \times \frac{9}{9} = \frac{9}{45}$	$3\frac{9}{45}$	
$+6\frac{4}{9}$	$+6\frac{4}{9}$	$\frac{4}{9} \times \frac{5}{5} = \frac{20}{45}$	$+6\frac{20}{45}$	$9\frac{29}{45}$
			$\frac{9}{9} \quad \frac{29}{45}$	

Step 1: Rewrite horizontal problems vertically. (This step is not necessary, but many students find it easier to add fractions when the problems are written vertically.)

Step 2: Separate the problem into addition of whole numbers and addition of fractions.

Step 3: Find a common denominator (a common multiple of the denominators of two or more fractions) for the fractions. The common denominator is 45 (because both 5 and 9 will divide into 45). Multiply $1/5$ by $9/9$. Multiply $4/9$ by $5/5$.

Step 4: Add the whole numbers ($3 + 6 = 9$). Add the numerators ($9 + 20 = 29$). The denominator remains the same (45).

Step 5: Combine the whole number and fraction to produce the answer.

Answer: $9\frac{29}{45}$

It may be necessary to reduce a fraction that is part of an answer. A fraction is in lowest terms when the numerator and denominator do not have a common factor greater than one. To reduce a fraction, determine the largest number that the numerator and the denominator can both be divided by and divide them by that number.

Example 2: Reduce all fractions to lowest terms.

$$\frac{32}{40}$$

$$\frac{32 \div 8}{40 \div 8} = \frac{4}{5}$$

Solution: The largest number 32 and 40 can both be divided by is 8. Divide 32 by 8 and divide 40 by 8.

Answer: $\frac{4}{5}$

Volume - C

Volume is the measurement of a three-dimensional figure's interior space. Volume is measured in cubic units.

The formula for calculating the volume of a rectangular solid is length multiplied by width multiplied by height.

$$\text{Volume} = \text{length} \times \text{width} \times \text{height}$$

Example: A figure has length = 2 inches, width = 4 inches, and height = 6 inches. Multiply the length, width, and height.

$$V = 2 \times 4 \times 6 = 48$$

Answer: 48 cubic inches.

It may be helpful to develop a series of problems and help the student determine the correct volume measurement.

Mass/Capacity - B

Mass is the total amount of matter that a figure contains. Capacity is the liquid volume of a figure.

A creative method for improving the student's understanding of measurement is to utilize actual objects. For example, help the student determine the approximate mass and capacity of objects in the home. For capacity, use actual figures, such as glasses and other containers. Help the student determine the capacity of liquid in each container.

	g = grams
1 milligram (mg)	= 0.001 g
1 centigram (cg)	= 0.01 g
1 decigram (dg)	= 0.1 g
1 dekagram (dag)	= 10 g
1 hectogram (hg)	= 100 g
1 kilogram (kg)	= 1,000 g
1 metric ton (t)	= 1,000 kg

The relationships in this chart are the same for liters and meters, except for the metric ton relationship.

Example: Solve.

$$24\text{g} = \underline{\quad} \text{mg}$$

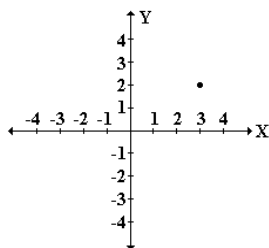
- A. 24,000
- B. 24
- C. 2,400
- D. 240,000

Since 1 gram equals 1,000 milligrams, 24 grams equal 24,000 milligrams. $24 \times 1,000 = 24,000$

Answer: A

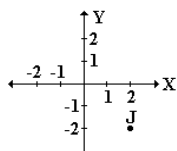
Coordinate Geometry - C

A coordinate graph is used to name the position of points. The x-coordinate (horizontal) is listed first and the y-coordinate (vertical) is listed second. For example, the coordinate pair (3, 2) is at the horizontal position 3 and the vertical position 2.



It may be helpful to use graph paper to develop a coordinate graph. Help the student plot points on the graph and determine the coordinate pair.

Example 1: What is the ordered pair for point J?



Answer: (2, -2) because the point J is 2 units over and 2 units down.

Order of Operations with Decimals - B

Performing operations with decimals is similar to performing operations with whole numbers.

Operations inside parentheses are performed first.

Example 1: $(2.3 \times 5.4) \div (1.3 \times 1.1) = ?$

(1) $2.3 \times 5.4 = 12.42$ and $1.3 \times 1.1 = 1.43$

(2) $12.42 \div 1.43 = ?$

(3) $12.42 \div 1.43 = 8.685314685$

Step 1: Perform all operations in parentheses. Multiply 2.3×5.4 . And multiply 1.3×1.1 .

Step 2: Rewrite the equation with the new numbers in place of the parentheses.

Step 3: Divide 12.42 by 1.43.

The answer is 8.685314685.

Example 2: $34.25 \div (1.23 \times 2.12) = ?$

(1) $1.23 \times 2.12 = 2.6076$

(2) $34.25 \div 2.6076 = ?$

(3) $34.25 \div 2.6076 = 13.13468323$

Step 1: Perform all operations in parentheses. Multiply 1.23×2.12 .

Step 2: Rewrite the equation with the new number in place of the parentheses.

Step 3: Divide 34.25 by 2.6076.

The answer is 8.685314685.

Units of Measurement - C

The metric system is a system of weights and measures in which the meter, the gram, and the liter are the basic units. Students should be able to convert between different metric measurements.

Units in the metric system can be converted using prefixes. The metric prefixes and their abbreviations are shown below.

milli = m

centi = c

deci = d

kilo = k

hecto = h

deka = da

These prefixes are added to base metric units, like the meter (m), liter (L), and gram (g), to denote different values. For instance, if the prefix kilo (k) is added to the base unit gram (g), it becomes kilogram (kg). A kilogram is 1,000 times larger than a gram, or in other words there are 1,000 grams in a kilogram. The following table shows the relationship between the metric units of length, volume, and weight.

Length (meters)	Volume (liters)	Weight (grams)
1 millimeter (mm) = 0.001 m	1 milliliter (mL) = 0.001 L	1 milligram (mg) = 0.001 g
1 centimeter (cm) = 0.01 m	1 centiliter (cL) = 0.01 L	1 centigram (cg) = 0.01 g
1 decimeter (dm) = 0.1 m	1 deciliter (dL) = 0.1 L	1 decigram (dg) = 0.1 g
meter (m)	liter (L)	grams (g)
1 dekameter (dam) = 10 m	1 dekaliter (daL) = 10 L	1 dekagram (dag) = 10 g
1 hectometer (hm) = 100 m	1 hectoliter (hL) = 100 L	1 hectogram (hg) = 100 g
1 kilometer (km) = 1,000 m	1 kiloliter (kL) = 1,000 L	1 kilogram (kg) = 1,000 g

To use this information in a conversion (for example, to convert 0.25 grams to milligrams), first determine how many grams are in 1 milligram. According to the chart, there are 0.001 grams in a milligram. Since the number of base units is known (0.25 grams) and the prefix unit (milligrams) needs to be found, divide 0.25 by 0.001, resulting in 250. There are 250 milligrams in 0.25 grams.

If the number of prefix units is known and the base unit needs to be found, multiply instead of divide. For example, to convert 650 milligrams to grams, we must multiply 650 by 0.001, resulting in 0.65. There are 0.65 grams in 650 milligrams.

Example 1: Solve.

$$9 \text{ dam} = \underline{\quad ? \quad} \text{ m}$$

Solution: According to the information in the chart above, 1 dekameter (dam) equals 10 meters (m). Since there are 9 dam, multiply 9 by 10 to get 90 m.

Answer: 90 m

Example 2: How many milliliters are there in 35 liters?

Solution: According to the information in the chart above, 1 milliliter (mL) equals 0.001 liters (L). Since the number of liters is known, divide 35 by 0.001 to determine the number of milliliters (35,000).

Answer: 35,000 mL

To help students understand conversions of metric units, have them measure various items with a metric ruler or meter stick. Once they find the length of each object, have them convert the measurement to another unit.

Volume - D

Volume is the measurement of a three-dimensional figure's interior space. Volume is measured in cubic units.

The formula for calculating the volume of a rectangular solid is length multiplied by width multiplied by height.

$$\text{Volume} = \text{length} \times \text{width} \times \text{height}$$

Example: A figure has length = 2 inches, width = 4 inches, and height = 6 inches. Multiply the length, width, and height.

$$V = 2 \times 4 \times 6 = 48$$

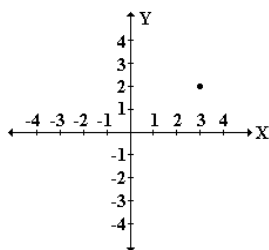
Answer: 48 cubic inches.

It may be helpful to develop a series of problems and help the student determine the correct volume measurement.

Coordinate Geometry - D

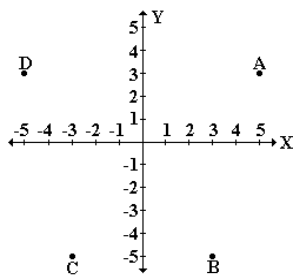
A coordinate graph is used to name the position of points. The x-coordinate (horizontal) is listed first and the y-coordinate (vertical) is listed second.

For example, the coordinate pair (3, 2) is at the horizontal position 3 and the vertical position 2.



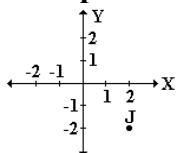
It may be helpful to use graph paper to develop a coordinate graph. Help the student plot points on the graph and determine the coordinate pair.

Example 1: What letter is at point (-5, 3)?



Answer: D because it is -5 units over and 3 units up.

Example 2: What is the ordered pair for point J?



Answer: (2, -2) because the point J is 2 units to the right on the x-axis and 2 units down on the y-axis.

Properties - D

Students must be able to solve for a missing value in a given equation. Understanding properties such as the order of operations is the key to correctly solving these problems.

Please review the following rules with the student:

1. Multiplication by 0: the product of any integer and 0 equals 0.

$$-3 \times 0 = 0 \qquad 3 \times 0 = 0$$

2. Associative Property of Addition: $(a + b) + c = a + (b + c)$.

$$(1 + 2) + 3 = 1 + (2 + 3)$$

3. Associative Property of Multiplication: $(a \times b) \times c = a \times (b \times c)$.

$$(1 \times 2) \times 3 = 1 \times (2 \times 3)$$

4. Reciprocals: two numbers are reciprocals if their product equals 1.

$$\frac{1}{3} \times \frac{3}{1} = 1 \quad \text{and} \quad \frac{2}{5} \times \frac{5}{2} = 1$$

5. Commutative Property of Addition: $a + b = b + a$

$$1 + 2 = 2 + 1$$

6. Commutative Property of Multiplication: $a \times b = b \times a$

$$1 \times 2 = 2 \times 1$$

7. Order of Operations:

A. When calculations for a given expression or equation require both addition and multiplication, the rule is to multiply first and add second.

$$(3)(2) + 3 = ?$$

$$6 + 3 = ?$$

$$6 + 3 = 9$$

B. The number outside the parentheses is multiplied with each number within the parentheses:

$$x(y + z) = xy + xz.$$

$$(1) \quad 3(x + y)$$

$$(2) \quad 3(x) + 3(y)$$

$$(3) \quad 3x + 3y$$

C. If a given expression contains both parentheses and brackets, calculations should be completed working from the innermost parentheses or bracket outward.

$$(1) \quad 5[4 + 3(x - 1 + 3)]$$

$$(2) \quad 5[4 + 3(x + 2)]$$

$$(3) \quad 5[4 + 3x + 6]$$

$$(4) \quad 5[10 + 3x]$$

$$(5) \quad 50 + 15x$$

The following are sample questions using the above properties.

Example 1: Which answer best completes the number sentence?

$$5_ = (5 \times 4) + (5 \times 6)$$

A. $x(4 + 6)$

B. $+(4 + 6)$

C. $x(20 + 30)$

D. $+(4 \times 6)$

Answer: A (because of rule 7B)

Example 2: Which one of the following best completes the number sentence?

$$(2.3 + 3.1) \times 5.6 = \underline{\quad ? \quad}$$

A. 3.5×5.6

B. 7.2×5.6

C. 9.1×5.6

D. 5.4×5.6

Answer: D (because of rule 7A)

Example 3: What is the value of n in the following statement?

$$13 \times (3.4 \times 0) = n$$

A. 44.2

B. 0

C. 13

D. 3.4

Answer: B (because of rule 1)

Units of Measurement - D

Students are given questions pertaining to units of length, weight, mass, and capacity for both the U.S. Standard (Customary) and the Metric System.

The following shows the relationship between the Customary Units of measure:

12 inches = 1 foot
3 feet = 1 yard
36 inches = 1 yard
5280 feet = 1 mile
1760 yards = 1 mile
4 quarts = 1 gallon
2 pints = 1 quart
2 cups = 1 pint
4 cups = 1 quart
16 ounces = 1 pound

The following shows the relationship between the Metric Units of length:

10 millimeters = 1 centimeter
100 millimeters = 1 decimeter
1000 millimeters = 1 meter
10 centimeters = 1 decimeter
100 centimeters = 1 meter
1000 meters = 1 kilometer
100 meters = 1 hectometer
10 meters = 1 dekameter

Example 1: A table is 36 inches tall. How tall is the table in feet?

Solution: There are 12 inches in 1 foot. Divide 36 by 12, the result is 3.

Answer: 3 feet

The student should also practice adding and subtracting different units of measurement.

Example 2: Solve.

$$\begin{array}{r} 2 \text{ ft} \quad 7 \text{ in} \\ - \quad \quad 10 \text{ in} \\ \hline \end{array}$$

$$\begin{array}{r} (1) \quad 1 \text{ ft} \quad 19 \text{ in} \\ \quad - \quad \quad 10 \text{ in} \\ \hline \end{array}$$

$$(2) \quad 1 \text{ ft} \quad 9 \text{ in}$$

Step 1: Borrow 1 ft (12 in) from the 2 feet, making the 2 a 1. Make the 7 a 19 ($7 + 12 = 19$).

Step 2: Subtract. $1 \text{ ft} - 0 \text{ ft} = 1 \text{ ft}$ and $19 \text{ in} - 10 \text{ in} = 9 \text{ in}$

Answer: 9 in

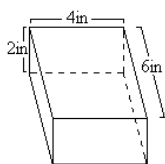
Volume of Rectangular Prisms

Volume is the measurement of a three-dimensional figure's interior space. Volume is measured in cubic units.

The formula for calculating volume of a rectangular prism is:

$$\text{Volume} = \text{length} \times \text{width} \times \text{height}$$

Example 1: Find the volume of a rectangular prism with length = 6 inches, width = 4 inches, height = 2 inches.



- (1) $\text{Volume} = 2 \times 4 \times 6$
- (2) $\text{Volume} = 48 \text{ cubic inches}$

Step 1: Apply the amounts given in the problem to the formula.

Step 2: Perform calculations to find the answer.

Answer: 48 cubic inches

Example 2: What is the height of the rectangular prism with volume = 10 cubic meters, length = 2 meters, and width = 1 meter?

- (1) $10 = (2)(1)(h)$
- (2) $10 = 2(h)$
- (3) $5 = h$

Step 1: Substitute the known values into the formula for the volume of a rectangular prism.

Step 2: Multiply 2 by 1 by h to get $2(h)$.

Step 3: Divide both sides of the equation by 2 to get that $h = 5$.

Answer: 5 meters

Spending

Many times there are items we wish to buy and we need to know how long we will have to save to buy these items, how much money we will have left after we purchase them, or how much money we will save by buying an item on sale. The following are examples of how to solve these types of problems.

Example 1: Carolyn wants to buy a new skateboard that costs \$230.00. Her grandmother said she will help Carolyn by paying for one-half of the skateboard. If Carolyn already has \$53.00 saved and she

makes \$23.00 per month babysitting, how many months will it take her to save the money to buy the skateboard?

- (1) $\$230.00 \div 2 = \115.00
- (2) $\$115.00 - \$53.00 = \$62.00$
- (3) $\$62.00 \div \$23.00 = 2.695652\dots$
- (4) 3 months

Step 1: Since Carolyn only has to pay for half of the skateboard, divide the price of the skateboard (\$230.00) by 2. Carolyn has to pay \$115.00 for the skateboard.

Step 2: Carolyn already has \$53.00 saved, so subtract \$53.00 from \$115.00 to determine the amount of money Carolyn still needs.

Step 3: Carolyn still needs \$62.00 for the skateboard and she makes \$23.00 per month. Dividing \$62.00 by \$23.00 will determine the number of months Carolyn will have to save in order to buy the skateboard.

Step 4: Since Carolyn will have to save for more than two months, it will be 3 months before she can buy the skateboard.

Answer: 3 months

Example 2: Colby went to a store where he bought a pair of tennis shoes that regularly cost \$96.99 and were 45% off and a sweatshirt for \$29.99 that was regularly \$52.49. He also bought a pair of athletic socks for \$5.99. (1) How much money did Colby spend? (2) How much money did Colby save?

Question #1:

- (1) $\$96.99 (45\%) = \text{Amount saved}$
- (2) $\$96.99 (0.45) = 43.6455$
- (3) $\$43.65 = \text{discount on shoes}$
- (4) $\$96.99 - \$43.65 = \$53.34$
- (5) $\$53.34 + \$29.99 + \$5.99 = \89.32

Step 1: To figure the amount of money saved on the shoes, multiply the original price of the shoes by the percent of discount.

Step 2: To multiply by 45%, first convert 45% into a decimal number. This involves moving the decimal point two places to the left.

$$45\% = 45.0\% = .45$$

Multiply \$96.99 by 0.45 to get 43.6455.

Step 3: Round 43.6455 to the nearest cent. This involves looking at the number in the hundredths place in 43.6455. The 5 to the right of the number in the hundredths place tells us to round the 4 up to 5. 43.6455 is rounded to \$43.65. Colby will save \$43.65 on the shoes.

Step 4: To determine the amount of money Colby will spend on the shoes, subtract \$43.65 from the original price of the shoes (\$96.99). Colby will pay \$53.34 for the shoes.

Step 5: Add the amount Colby spent on the shoes (\$53.34) to the amount he spent on the sweatshirt (\$29.99) and the amount he spent on the socks (\$5.99).

Answer: Colby spent a total of \$89.32.

Question #2:

$$(6) \$52.49 - \$29.99 = \$22.50$$

$$(7) \$43.65 + \$22.50 = \$66.15$$

Step 6: Determine the amount of money Colby saved on the sweatshirt. Subtract the amount Colby paid for the sweatshirt (\$29.99) from the regular price of the sweatshirt (\$52.49). Colby saved \$22.50 on the sweatshirt.

Step 7: Determine the total amount of money saved. Add the amount of money Colby saved on the shoes (\$43.65) to the amount of money Colby saved on the sweatshirt (\$22.50).

Answer: Colby saved a total of \$66.15.

Example 3: A sweater that costs \$85.00 is on sale for 25% off. Katie has \$69.00. If sales tax is 7.25%, how much money will Katie have left over after she buys the sweater?

$$(1) \$85.00 (0.25) = \$21.25$$

$$(2) \$85.00 - \$21.25 = \$63.75$$

$$(3) \$63.75 (0.0725) = 4.621875 \sim \$4.62$$

$$(4) \$63.75 + \$4.62 = \$68.37$$

$$(5) \$69.00 - \$68.37 = \$0.63$$

Step 1: Determine the amount of money saved on the sweater by multiplying \$85.00 by 25%. Remember to change 25% into a decimal. Katie saves \$21.25 on the sweater.

Step 2: Determine the amount Katie will pay for the sweater by subtracting the amount saved (\$21.25) from the original price of the sweater (\$85.00). Katie pays \$63.75 for the sweater.

Step 3: Figure the sales tax on the purchase by multiplying the purchase price of the sweater (\$63.75) by 7.25%. Remember to convert 7.25% into a decimal (0.0725). The sales tax will be \$4.62. The $\sim$ symbol means approximately.

Step 4: Add the purchase price of the sweater (\$63.75) and the sales tax (\$4.62). The total price of the sweater is \$68.37.

Step 5: Katie has \$69.00 and we want to know the amount of money she will have left over after she buys the sweater. Subtract \$68.37 from \$69.00.

Answer: Katie will have \$0.63 left over after she buys the sweater.

Example 5: Tanner had \$42.00. He spent $\frac{1}{4}$ of his money on tickets to the dance and $\frac{3}{5}$ of his money on a corsage for his date. How much money, if any, does Tanner have left?

$$(1) \$42.00 \div 4 = \$10.50$$

$$(2) \$42.00 \div 5 = \$8.40$$

$$(3) \$8.40 \times 3 = \$25.20$$

$$(4) \$10.50 + \$25.20 = \$35.70$$

$$(5) \$42.00 - \$35.70 = \$6.30$$

Step 1: Determine the amount of money Tanner spent on tickets to the dance. To figure one-fourth of a number, divide the number by 4. Tanner spent \$10.50 on the tickets.

Step 2: Determine the amount of money Tanner spent on the corsage. To figure one-fifth of a number, divide the number by 5. One-fifth of \$42.00 is \$8.40.

Step 3: Tanner spent three-fifths of his money on the corsage, so we need to multiply \$8.40 by 3. The corsage cost \$25.20.

Step 4: Determine the total amount of money Tanner spent. Add the price of the tickets (\$10.50) and the

cost of the corsage (\$25.20). Tanner spent \$35.70.

Step 5: To determine the amount of money Tanner has left, subtract \$35.70 from \$42.00.

Answer: Tanner has \$6.30 left over.

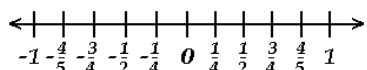
Addition/Subtraction Rational Numbers

The following numbers are rational numbers because they can all be written as fractions.

$$\begin{array}{ll} 0.6 = \frac{3}{5} & 0 = \frac{0}{1} \\ -29 = \frac{-29}{1} & 2 = \frac{2}{1} \end{array}$$

Adding and subtracting rational numbers includes the calculation of whole numbers, fractions, decimals, and integers.

The most common error that students make when learning how to add and subtract rational numbers is incorrectly ordering the rational numbers. A useful technique for showing the student how to identify rational numbers is to create a number line. Negative numbers appear to the left of 0, while positive numbers appear to the right. The following is an example of a number line:



Confirm that the student understands that for any two rational numbers on the number line, the number to the right is always the greater number. For example, $1/4$ is greater than $-1/4$. When he or she understands how to order rational numbers, have him or her add $1/4$ to $-1/4$. The answer is 0 as shown in the number line above because $-1/4$ is $1/4$ places from 0 on the number line.

Example 1: $-3.2 + -5.7 - -4.3 = ?$

$$(1) -3.2 + -5.7 = -8.9$$

$$(2) -8.9 - -4.3 = -8.9 + 4.3 = -4.6$$

Step 1: Add or subtract the first two numbers from left to right. Remember that when we add two negative numbers, we add their absolute values together and give the result a negative sign.

Step 2: Replace $-3.2 + -5.7$ with -8.9 and continue adding or subtracting left to right. Remember that when we subtract a negative, it's the same as adding a positive, so we can rewrite $-8.9 - -4.3 = ?$, as $-8.9 + 4.3 = ?$

Answer: -4.6

Example 2: $2 \frac{1}{2} - 3 \frac{3}{4} + -5 \frac{5}{8} =$

There are several ways to solve this problem with fractions, one method is to change the fractions to improper fractions:

$$2 \frac{1}{2} = \frac{5}{2}$$

$$3 \frac{3}{4} = \frac{15}{4}$$

$$5 \frac{5}{8} = \frac{45}{8}$$

Then we can find a common denominator and perform the addition and subtraction. The common denominator is 8, so we make each denominator 8:

$$5/2 = 20/8$$

$$15/4 = 30/8$$

$$45/8 = 45/8$$

Now we can rewrite the problem as:

$$20/8 - 30/8 + 45/8 = ?$$

$$(1) 20/8 - 30/8 = 20/8 + 30/8 = 50/8$$

$$(2) 50/8 + 45/8 = 5/8$$

Step 1: Add or subtract the first pair of fractions from left to right.

Step 2: Replace $20/8 - 30/8$ with $50/8$ and continue adding or subtracting from left to right.

Answer: $5/8$

Mass/Capacity - C

Mass is the total amount of matter that a figure contains. Capacity is the liquid content or volume of a figure. Mass and capacity are communicated using the metric system. Mass/capacity problems require students to estimate the mass or capacity of specific amounts and to convert grams to milligrams, metric tons to grams, etc.

Before the student can solve mass and capacity problems, he or she must first understand mass/capacity metric measurements. The gram is the basis of weight measurements in the metric system. Here is a basic breakdown of the metric system of weight.

1,000 milligrams (mg) = 1 gram (g)
100centigrams (cg) = 1 gram (g)
10 decigrams (dg) = 1 gram (g)
1 dekagram (dag) = 10 grams (g)
1 hectogram (hg) = 100 grams (g)
1 kilogram (kg) = 1,000 grams (g)
1 metric ton (t) = 1,000,000 grams

The liter is the basis of capacity measurements in the metric system. Here is a basic breakdown of the metric system of capacity.

1,000 milliliters (ml) = 1 liter (l)
100centiliters (cl) = 1 liter (l)
10 deciliters (dl) = 1 liter (l)
1 dekaliter (dal) = 10 liters (l)
1 hectoliter (hl) = 100 liters (l)
1 kiloliter (kl) = 1,000 liters (l)

When the student understands these conversions, he or she is ready to make estimations. To help the student determine the approximate mass, give him or her objects from around the house to make estimates. A loaf of bread, for instance, probably has a mass of around 500 grams. For capacity questions, use glasses and other containers. Help the student estimate the capacity of liquid in each container.

Length - B

Students are required to convert units of measurement.

The following shows the relationship between the Customary Units of length:

12 inches = 1 foot
3 feet = 1 yard
12 inches = 1 foot
36 inches = 1 yard
5280 feet = 1 mile
1760 yards = 1 mile

The following shows the relationship between the Metric Units of length:

10 millimeters = 1 centimeter
100 millimeters = 1 decimeter
1000 millimeters = 1 meter
10 centimeters = 1 decimeter
100 centimeters = 1 meter
1000 meters = 1 kilometer
100 meters = 1 hectometer
10 meters = 1 dekameter

Example 1: A table is 36 inches tall. There are 12 inches in 1 foot. Divide 36 by 12, the result is 3. The table is 3 feet tall. Continue with other objects and also use the metric system.

Example 2: A fence is 4 meters long. Jim put a gate on the end of the fence that was 20 decimeters long. How long is the fence and the gate combined?

- (1) 20 decimeters = 2 meters
- (2) 4 meters + 2 meters = ?

Step 1: Convert units to like units.

Step 2: Add the units.

Answer: 6 meters